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EVALUATION OF SOME RELIABILITY METRICS OF A STANDBY THREE-UNIT SYSTEM

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ABSTRACT

Reliability evaluation of standby redundant systems has become a critical aspect of engineering design, especially in applications where continuous operation is required. This study focuses on a three-unit standby system, in which two identical units operate simultaneously while the third remains in standby to improve fault tolerance. The objective is to evaluate key reliability metrics such as Steady state availability, Busy period, profit and mean time to failure (MTTF). To achieve this, a continuous-time Markov chain (CTMC) approach is employed, incorporating the system's transition dynamics under both active and standby configurations. Explicit expressions are derived for the reliability indices, and numerical illustrations are provided to highlight the system's performance under varying failure and repair rates. The results demonstrate that the standby unit significantly enhances the system's dependability by prolonging operational lifetime and improving availability compared to non-redundant configurations. The study concludes that such a configuration is suitable for mission-critical applications where downtime must be minimized. The findings provide valuable insights for system designers, particularly in optimizing redundancy and repair strategies for cost-effective and resilient engineering systems.

Key word; Availability, Busy period, Mean time failure, And Reliability.

1.0 INTRODUCTION

Reliability analysis plays a vital role in the design and evaluation of engineering systems, particularly in domains where uninterrupted operation is critical, such as power generation, telecommunication networks, and industrial automation. To achieve higher fault tolerance and extended operational lifetime, redundancy techniques such as standby configurations are widely applied [1–3]. In a standby system, a subset of units is kept in reserve and activated only when an active unit fails. This approach balances cost-effectiveness and reliability, ensuring that system resources are not wasted while still providing fault coverage. Among redundancy strategies, multi-unit standby systems have gained increasing attention because of their ability to enhance system dependability without the high operational costs associated with fully parallel redundancy [4]. A three-unit standby system, where two units operate concurrently and one remains in standby, represents a practical configuration that is often applied in mission-critical engineering environments. Evaluating its performance requires robust modeling of system reliability metrics such as the reliability function, availability, mean time to failure (MTTF), and steady-state behavior [5–7]. Despite considerable work on k-out-of-n and standby redundancy models, several research gaps remain. First, much of the existing literature focuses on either pure parallel or series systems, with fewer studies dedicated to hybrid active-standby configurations [8]. Second, while reliability metrics have been studied extensively, fewer works have simultaneously examined multiple indices such as reliability, availability, and MTTF in a unified framework [9]. Third, practical implications for system designers—particularly how standby redundancy impacts cost effectiveness and operational resilience are often underexplored [10]. In high availability engineering systems such as telecommunications infrastructure, power-generation plants, and aerospace control platforms redundancy in the form of standby units plays a crucial role in ensuring continuous operation when primary components fail. A typical design in this context is the three unit standby configuration, where one or more units remain in reserve (standby) to replace failing active units. In such systems, key reliability metrics such as mean time to failure (MTTF), busy period, profit, and steady-state availability are essential for system design, performance optimization, and maintenance planning. Between 2021 and 2025, research into standby redundancy systems has evolved to address increasingly realistic and complex conditions. Roy and Gupta (2021) investigated the reliability of a k-out-of-n system with a cold standby component, using copula-based lifetime dependency models, highlighting early modern efforts in modeling dependence among components [11]. Wang et al. (2023) introduced a survival-signature-based framework for reliability evaluation of standby redundant systems, allowing for general lifetime distributions (e.g., Weibull, log-normal) and applying sample-based methods to reduce the computational burden in large state spaces [12]. Karadayi and Iscioglu (2023) proposed a mean residual life (MRL) based assessment method tailored for multi-state standby systems, moving beyond traditional binary models [13]. More recently, Kanta and Chaudhary (2024) analyzed the availability of warm standby systems under fault detection delays and general repair distributions, incorporating *non-ideal switching and imperfect fault detection mechanisms [14]. These recent developments form the foundation for advanced modeling of standby systems with greater applicability to real-world engineering problems, including those with more than two active/standby units, delayed switching, and non-exponential failure behaviors. This study addresses these gaps by evaluating the reliability metrics of a three-unit standby system consisting of identical units, two of which operate while the third is in standby. The objectives of this research are threefold:

1. To formulate a mathematical model of the system using a continuous-time Markov chain (CTMC) approach, capturing both active and standby transitions.
2. To derive explict expressions for key reliability indices, including the steady state availability, busy period , profit and mean time to failure.
3. To perform numerical evaluation of the system’s performance under varying failure and repair rates, highlighting its practical dependability.

The contributions of this study are summarized as follows:

1. Development of a CTMC-based framework that jointly evaluates multiple reliability measures for a standby system with mixed active and standby units.
2. Analytical derivation of closed-form expressions for reliability metrics, enhancing interpretability and enabling comparative evaluation.
3. Provision of numerical results that demonstrate the operational benefits of standby redundancy, offering actionable insights for system designers in mission-critical applications.

2.0 SYSTEM DESCRIPTION

The system under study consists of three identical units, A,B and C out of which two are active while the third remains on standby. At system initiation, both active units function simultaneously to provide service, while the standby unit is idle but available. In the event of a failure in any active unit, the standby component is switched into operation, thereby maintaining the required number of operational units. Repair facilities are assumed to be available, and failed units undergo corrective repair before being restored to either active or standby status.

2.1 States of the System

S1 : Initial state, Unit A and B are working while Unit C is on standby, the system is working
S2 : Components B and C are working and A is failed; the system is working.
S3 : Components A and C are working and B is failed; the system is working.
S4 : Component B is working and A and C are failed; the system is working.
S5 : Component C is working and A and B are failed; the system is working.
S6 : Component A is working and B and C are failed; the system is working.
S7 : Components A, B, C are failed; the system is failed.

3.0 MODEL FORMULATION

3.1 Model Assumptions

To facilitate tractable analysis, the following assumptions are adopted:

1. Failure and repair times of the units are statistically independent and follow exponential distributions.
2. The standby unit is a cold standby, meaning it does not fail while idle.
3. Repair of failed units is conducted by a single repair facility operating under first-comefirst-served discipline. The system consist of three identical units A,B, and C, A and B are working and C is on standby.
4. The system works, if atleast one component is working.
5. The system failed, if all the Units failed.
6. Standby activation occurs instantly upon the failure of an active unit, thereby moving the system into a new operational state.

3.2 Derivation of Reliability Metrics

Using the CTMC model, a system of first order ordinary differential equations is derived using

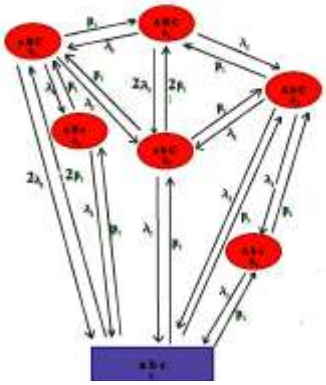

Figure 1: Transition Diagram of the System

$$\frac{dP_1}{dt} = -4\lambda_1 P_1(t) + \beta_1 P_2(t) + \beta_1 P_3(t) + 2\beta_1 P_5(t)$$

$$\frac{dP_2(t)}{dt} = -(\beta_1 + 4\lambda_1)P_2(t) + \lambda_1 P_1(t) + \beta_1 P_4(t) + \beta_1 P_5(t) + 2\beta_1 P_7(t)$$

$$\frac{dP_3(t)}{dt} = -(\beta_1 + 3\lambda_1)P_3(t) + \lambda_1 P_1(t) + \beta_1 P_5(t) + \beta_1 P_6(t) + \beta_1 P_7(t)$$

$$\frac{dP_4(t)}{dt} = -(\beta_1 + \lambda_1)P_4(t) + \lambda_1 P_2(t) + \beta_1 P_7(t)$$

$$\frac{dP_5(t)}{dt} = -(4\beta_1 + \lambda_1)P_5(t) + 2\lambda_1 P_1(t) + \lambda_1 P_2(t) + \lambda_1 P_3(t) + \beta_1 P_7(t) \qquad (1)$$

$$\frac{dP_6(t)}{dt} = -(\beta_1 + \lambda_1)P_6(t) + \lambda_1 P_3(t) + \beta_1 P_7(t)$$

$$\frac{dP_7(t)}{dt} = -6\beta_1 P_7(t) + 2\lambda_1 P_2(t) + \lambda_1 P_3(t) + \lambda_1 P_4(t) + \lambda_1 P_5(t) + \lambda_1 P_6(t)$$

With initial condition

$$P(0) = [1, 0, 0, 0, 0, 0, 0], \qquad (2)$$

And the system can be written in matrix form as:

$$\dot{P} = AP,$$

where;

$$\dot{P} = \begin{bmatrix} P_1'(t) \\ P_2'(t) \\ P_3'(t) \\ P_4'(t) \\ P_5'(t) \\ P_6'(t) \\ P_7'(t) \end{bmatrix}, \quad P = \begin{bmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \\ P_4(t) \\ P_5(t) \\ P_6(t) \\ P_7(t) \end{bmatrix},$$

and the 7 × 7 transition rate matrix A is given by:

$$A = \begin{bmatrix} -4\lambda_1 & \beta_1 & \beta_1 & 0 & 2\beta_1 & 0 & 0 \\ -\lambda_1 & -(\beta_1 + 4\lambda_1) & 0 & \beta_1 & \beta_1 & 0 & 2\beta_1 \\ \lambda_1 & 0 & -(\beta_1 + 3\lambda_1) & 0 & \beta_1 & \beta_1 & \beta_1 \\ 0 & \lambda_1 & 0 & -(\beta_1 + \lambda_1) & 0 & 0 & \beta_1 \\ 2\lambda_1 & \lambda_1 & \lambda_1 & 0 & -(4\beta_1 + \lambda_1) & 0 & \beta_1 \\ 0 & 0 & \lambda_1 & 0 & 0 & -(\beta_1 + \lambda_1) & \beta_1 \\ & 0 & 2\lambda_1 & \lambda_1 & \lambda_1 & \lambda_1 & -6\beta_1 \end{bmatrix} \qquad (3)$$

3.3 Steady State Availability

To analyze the availability of a system with different states, we use the initial condition in (1) above. The system can be expressed as:

$$\dot{P} = AP$$

where

$$A=\begin{bmatrix} -4\lambda_1 & \beta_1 & \beta_1 & 0 & 2\beta_1 & 0 & 0 \\ -\lambda_1 & -(\beta_1+4\lambda_1) & 0 & \beta_1 & \beta_1 & 0 & 2\beta_1 \\ \lambda_1 & 0 & -(\beta_1+3\lambda_1) & 0 & \beta_1 & \beta_1 & \beta_1 \\ 0 & \lambda_1 & 0 & -(\beta_1+\lambda_1) & 0 & 0 & \beta_1 \\ 2\lambda_1 & \lambda_1 & \lambda_1 & 0 & -(4\beta_1+\lambda_1) & 0 & \beta_1 \\ 0 & 0 & \lambda_1 & 0 & 0 & -(\beta_1+\lambda_1) & \beta_1 \\ & 0 & 2\lambda_1 & \lambda_1 & \lambda_1 & \lambda_1 & -6\beta_1 \end{bmatrix}$$

with the following diagonal elements:

$q_{11} = 4\lambda_1$, $q_{22} = 4\lambda_1 + \beta_1$, $q_{33} = 3\lambda_1 + \beta_1$, $q_{44} = \beta_1 + \lambda_1$, $q_{55} = 4\beta_1 + \lambda_1$, $q_{66} = \lambda_1 + \beta_1$, $q_{77} = 6\beta_1$

The states S_1, S_2, S_3, S_4, S_5, S_6 are the only working states of the system. Thus, the steady-state availability of the system is the sum of the probabilities of these operating states.

$$A(\infty) = P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) + P_5(\infty) + P_6(\infty) \tag{3}$$

In the steady state, the derivative of the state probabilities becomes zero, and therefore the system of equations becomes:

$$A(\infty) = 0$$

Which in matrix form:

$$\begin{bmatrix} q_{11} & \beta_1 & \beta_1 & 0 & 2\beta_1 & 0 & 0 \\ -\lambda_1 & -q_{22} & 0 & \beta_1 & \beta_1 & 0 & 2\beta_1 \\ \lambda_1 & 0 & -q_{33} & 0 & \beta_1 & \beta_1 & \beta_1 \\ 0 & \lambda_1 & 0 & -q_{44} & 0 & 0 & \beta_1 \\ 2\lambda_1 & \lambda_1 & \lambda_1 & 0 & -q_{55} & 0 & \beta_1 \\ 0 & 0 & \lambda_1 & 0 & 0 & -q_{66} & \beta_1 \\ & 0 & 2\lambda_1 & \lambda_1 & \lambda_1 & \lambda_1 & -q_{77} \end{bmatrix} \begin{bmatrix} P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{4}$$

Using the following normalizing condition:

$$P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) + P_5(\infty) + P_6(\infty) + P_7(\infty) = 1 \tag{5}$$

We substitute (5) in the redundant row (last row) of (4) to get:

$$\begin{bmatrix} q_{11} & \beta_1 & \beta_1 & 0 & 2\beta_1 & 0 & 0 \\ -\lambda_1 & -q_{22} & 0 & \beta_1 & \beta_1 & 0 & 2\beta_1 \\ \lambda_1 & 0 & -q_{33} & 0 & \beta_1 & \beta_1 & \beta_1 \\ 0 & \lambda_1 & 0 & -q_{44} & 0 & 0 & \beta_1 \\ 2\lambda_1 & \lambda_1 & \lambda_1 & 0 & -q_{55} & 0 & \beta_1 \\ 0 & 0 & \lambda_1 & 0 & 0 & -q_{66} & \beta_1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{5}$$

And solve for $P_1(\infty),\ P_2(\infty),\ P_3(\infty),\ P_4(\infty),\ P_5(\infty), P_6(\infty)$, and $P_7(\infty)$.
The steady-state availability is given by,

$$A(\infty) \ = \ \frac{\beta_1\left(148\beta_1^4 + 525\beta_1^3\lambda_1 + 774\beta_1^2\lambda_1^2 + 494\beta_1^1\lambda_1^3 + 69\lambda_1^4\right)}{164\beta_1^5 + 569\beta_1^4\lambda_1 + 820\beta_1^3\lambda_1^2 + 492\beta_1^2\lambda_1^3 + 53\beta_1^1\lambda_1^4 - 12\lambda_1^5} \tag{6}$$

3.4 Busy Period Analysis

Here we use the same initial condition as in Figure (1).

3.4.1 For Busy Period 1

From states 2 to 6, the repairman is in the busy period due to minor failure $\beta_1(\infty)$, repairing the failed units.

The steady-state $B_1(\infty)$ is therefore:

$$B_1(\infty),\ =\ P_2(\infty) +\ P_3(\infty) +\ P_4(\infty) +\ P_5(\infty),\, P_6(\infty) \tag{7}$$

Thus,

$$\beta_1(\infty) \ = - \ \frac{2\beta_1\left(62\beta_1^4 + 220\beta_1^3\lambda_1 + 319\beta_1^2\lambda_1^2 + 198\beta_1\lambda_1^3 + 21\lambda_1^4\right)}{164\beta_1^5 + 569\beta_1^4\lambda_1 + 820\beta_1^3\lambda_1^2 + 492\beta_1^2\lambda_1^3 + 53\beta_1^1\lambda_1^4 - 12\lambda_1^5} \tag{8}$$

3.4.2 For Busy Period 2

The last state is the repairman’s busy period due to major failure $\beta_2(\infty)$

The steady-state $B_1(\infty)$, is;

$$B_2(\infty) = P_7(\infty), \tag{9}$$

$$\beta_2(\infty) \ = - \ \frac{2\left(-8\beta_1^5 - 22\beta_1^4\lambda_1\ 23\beta_1^3\lambda_1^2 + \beta_1^2\lambda_1^3 + 8\beta_1\lambda_1^4 + 6\lambda_1^5\right)}{164\beta_1^5 + 569\beta_1^4\lambda_1 + 820\beta_1^3\lambda_1^2 + 492\beta_1^2\lambda_1^3 + 53\beta_1^1\lambda_1^4 - 12\lambda_1^5} \tag{10}$$

3.5 Profit Analysis

The failed units are subjected to corrective maintenance, as can be observed in states S_1 to S_7. The repairmen are busy in those states performing corrective maintenance on the failed units.

Let C_0 and C_1 be the revenue generated when the system is in the working state and the income lost (or cost incurred) when in a failed state due to minor or complete failure (corrective maintenance).

Profit = Total Revenue when the system is working−Cost of maintenance when the system has minor or complete failure
Thus,

$$\text{Profit} = C_0 A\ (\infty) - \ C_1 \ .\ B_1(\infty) - \ C_2 \ .\ B_2(\infty) \tag{11}$$

3.6 Mean Time to System Failure Analysis

From Figure (1), let $P_i(t)$ be the probability that the system at time $t \geq 0$ is in state S_i. Also, let $P(t)$ be the probability row vector at time t; we use the following initial condition:

$$P(0) = \ [P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0), P_7(0)] \ = \ [1, 0, 0, 0, 0, 0, 0]$$

Equation (1) can be written in matrix form as:

$$\dot{P} = AP$$

where

$$A = \begin{bmatrix} -4\lambda_1 & \beta_1 & \beta_1 & 0 & 2\beta_1 & 0 & 0 \\ -\lambda_1 & -(\beta_1 + 4\lambda_1) & 0 & \beta_1 & \beta_1 & 0 & 2\beta_1 \\ \lambda_1 & 0 & -(\beta_1 + 3\lambda_1) & 0 & \beta_1 & \beta_1 & \beta_1 \\ 0 & \lambda_1 & 0 & -(\beta_1 + \lambda_1) & 0 & 0 & \beta_1 \\ 2\lambda_1 & \lambda_1 & \lambda_1 & 0 & -(4\beta_1 + \lambda_1) & 0 & \beta_1 \\ 0 & 0 & \lambda_1 & 0 & 0 & -(\beta_1 + \lambda_1) & \beta_1 \\ 0 & 2\lambda_1 & 2\lambda_1 & \lambda_1 & \lambda_1 & \lambda_1 & -6\beta_1 \end{bmatrix}$$

It is difficult to evaluate the transient solution directly due to the absorbing state. Hence, we delete the rows and columns corresponding to the absorbing state and take the transpose of the remaining submatrix A to produce a new matrix, say Q.

The expected time to reach an absorbing state is obtained from the following:

$$\mathbb{E}\left[T_{P(0)} \ \rightarrow P(absorbing)\right] \ = P(0) \int_0^{\infty} e^{Qt}\, dt \tag{12}$$

and since $Q^{-1} \ < \ 0,$

$$\int_0^{\infty} e^{Qt}\, dt \ \ = \ \ -Q^{-1} \tag{13}$$

Thus, the explicit expression for the Mean Time to System Failure (MTSF) is given by:

$$\mathbb{E}\left[T_{P(0)} \ \rightarrow P(absorbing)\right] \ = MTSF = p(0)\ (-Q^{-1}) \ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{14}$$

where,

$$Q=\begin{bmatrix} -4\lambda_1 & \lambda_1 & \lambda_1 & 0 & 2\lambda_1 & 0 \\ \beta_1 & -(\beta_1+4\lambda_1) & 0 & \lambda_1 & \lambda_1 & 0 \\ \beta_1 & 0 & -(\beta_1+\lambda_1) & 0 & \lambda_1 & \lambda_1 \\ 0 & \beta_1 & 0 & -(\beta_1+\lambda_1) & 0 & 0 \\ 2\lambda_1 & \beta_1 & \beta_1 & 0 & -(4\beta_1+\lambda_1) & 0 \\ 0 & 0 & \beta_1 & 0 & 0 & -(\beta_1+\lambda_1) \end{bmatrix}$$

Therefore, the explicit expression for the mean time to system failure is given by:

$$MTSF \;=\; \frac{4\beta_1^5+41\beta_1^4\lambda_1+158\beta_1^2\lambda_1^2+28\beta_1^2\lambda_1^3+232\beta_1\lambda_1^4+57\lambda_1^5}{\lambda_1^2\left(20\beta_1^4+109\beta_1^3\lambda_1+224\beta_1^2\lambda_1^2+191\beta_1\lambda_1^3+48\lambda_1^4\right)} \qquad (15)$$

4.0 RESULTS AND DISCUSSION

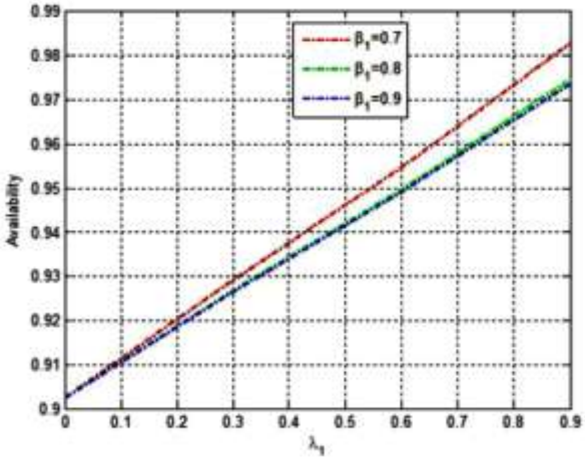

Figure 2. Availability against λ_1 for different values of β_1

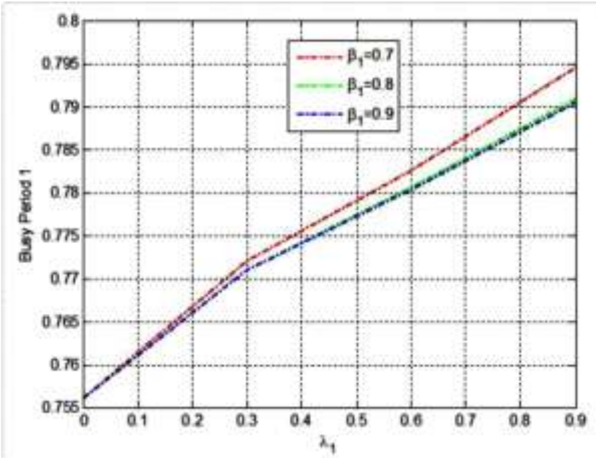

Figure 3. Busy Period against λ_1 for different values β_1

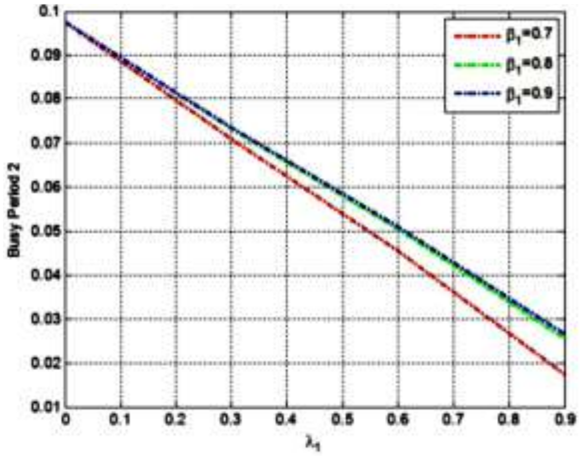

Figure 4. Busy Period 2 against λ_1 for different values β_1

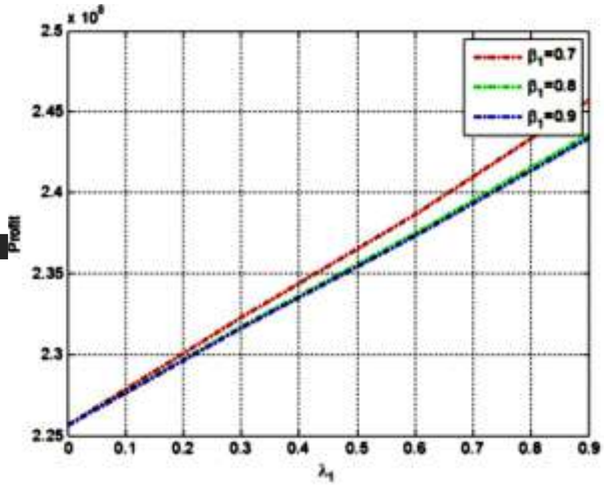

Figure 5. Profit against λ_1 for different values β_1

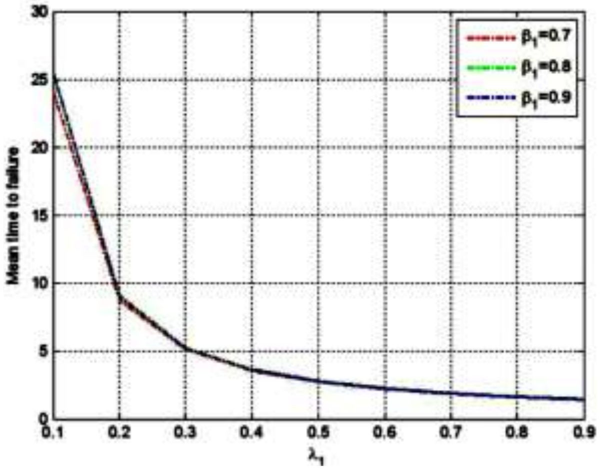

Figure 6. Mean time to failure against λ_1 for different values β_1

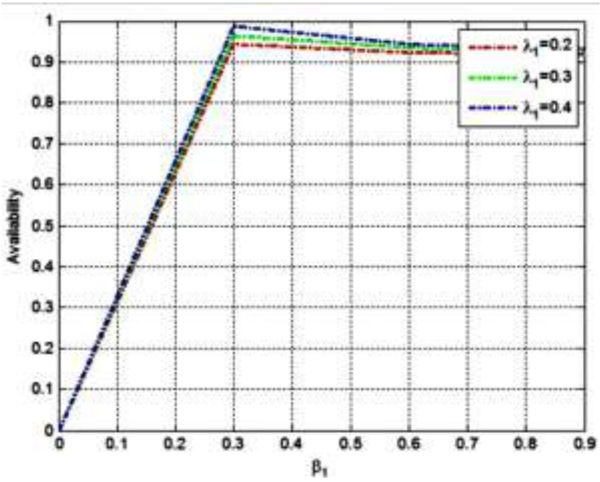

Figure 7. Availability against β_1 for different values λ_1

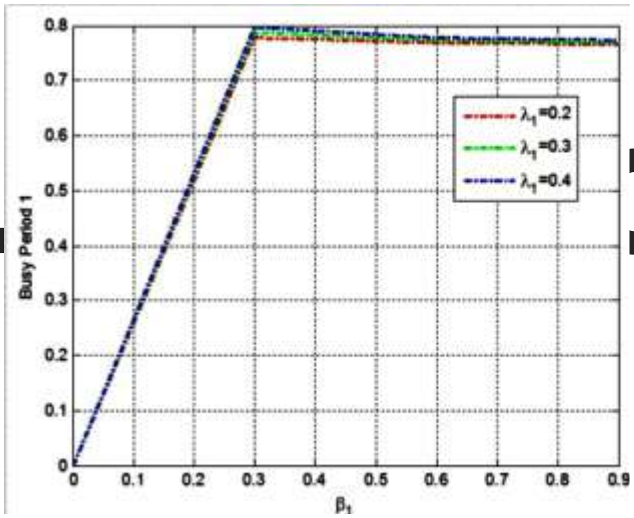

Figure 8. Busy Period 1 against β_1 for different values λ_1

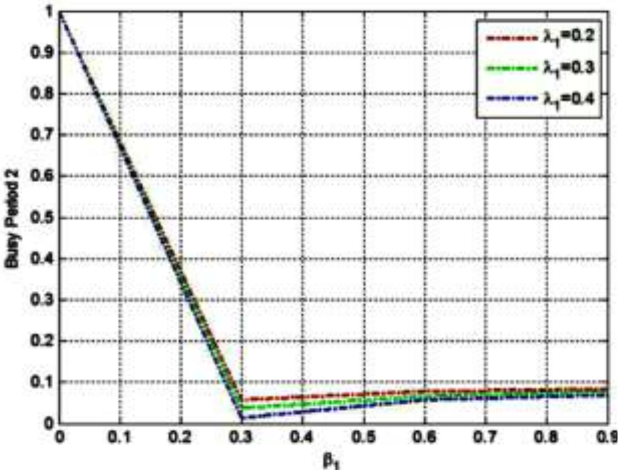

Figure 9. Busy Period 2 against β_1 for different values λ_1

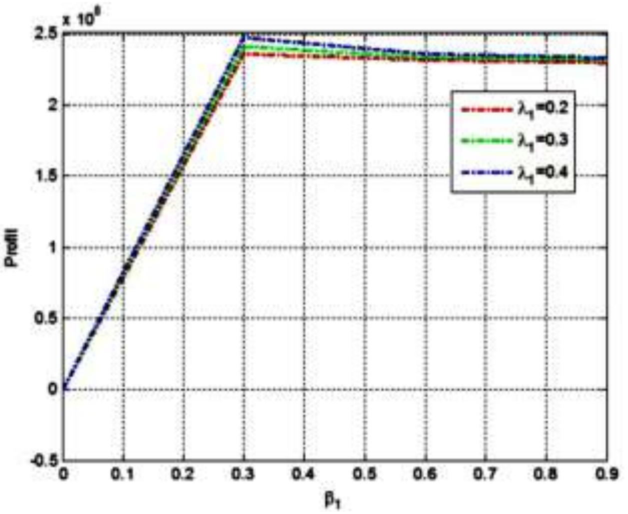

Figure 10. Profit against β_1 for different values λ_1

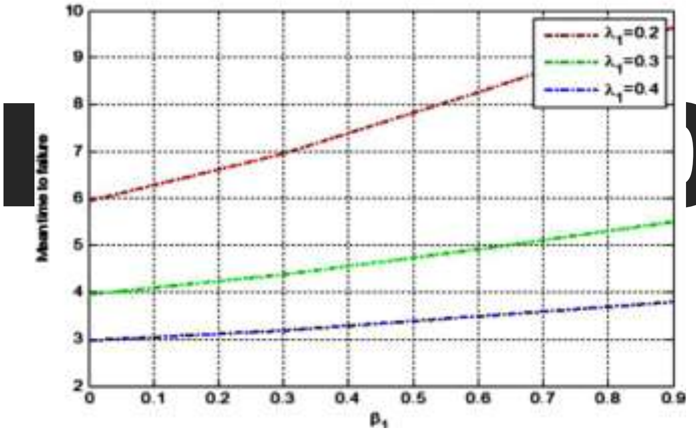

Figure 11. Profit against β_1 for different values λ_1

The simulation results presented in Figures below, figures 2, 3, and 6 illustrate the effect of the failure rate on system availability, profit, and mean time to failure (MTTF) under different repair rate conditions (β_1=0.7 ,0.8, 0.9). A consistent trend is observed: as the failure rate increases from 0 to 0.9, all three metrics—availability, profit, and MTTF—exhibit a progressive decline. This behavior is expected, as higher failure intensities reduce the likelihood of the system remaining operational, thereby shortening its effective lifetime and diminishing its economic returns. Importantly, the magnitude of these declines is strongly influenced by the repair rate. Across all performance indices, the system consistently achieves superior outcomes when the repair rate is high (β_1=0.9) compared to lower repair rates (β_1=0.8 and β_1=0.7). This demonstrates the stabilizing role of efficient repair mechanisms, which can substantially mitigate the adverse impact of frequent failures. In practical terms, investing in faster or more effective repair facilities enhances both the technical reliability and the economic viability of the system, underscoring the importance of repair rate optimization in system design.

Figures 4 and 5 further complement these insights by examining the behavior of busy periods of the repairman (Busy Period 1 and Busy Period 2). The results reveal that as the failure rate increases, both busy periods become longer, reflecting increased repair workload and resource utilization. However, the busy periods are noticeably shorter when the repair rate is higher (β_1=0.9), compared to moderate or low repair rates. This suggests that while frequent failures increase the burden on repair resources, higher repair efficiency not only reduces downtime but also prevents excessive congestion in repair queues. Collectively, these findings highlight a key trade-off in system reliability management: while failure rates are largely dictated by component design and environmental conditions, the repair rate is a controllable factor that can be strategically adjusted to enhance overall system performance. The study demonstrates that higher repair rates improve availability, profitability, and MTTF, while simultaneously reducing repairman busy periods, thereby offering both operational and economic benefits. From a broader perspective, the results emphasize the critical role of repair efficiency in maintaining sustainable system operation under uncertain failure environments. For real-world applications—such as power systems, communication networks, or irrigation pumping systems—the implication is clear: proactive investment in repair infrastructure and skilled maintenance personnel can yield significant long-term benefits in terms of reliability, cost-effectiveness, and resilience.

The simulation results presented in Figures 7, 8, and 9 illustrate the influence of the repair rate (β_1) on availability, profit, and mean time to failure (MTTF) under different values of the failure rate (λ_1 = 0.7, 0.8, 0.9). A consistent trend emerges: as the repair rate increases from 0 to 0.9, all three metrics improve significantly. This result is intuitive, as higher repair rates reduce downtime, restore units to operation more quickly, and thereby enhance the system's ability to sustain continuous service and generate profit. However, the relative performance across different failure rates shows important distinctions. For all repair rate values, the system achieves higher availability, profit, and MTTF at lower failure rates (λ_1 = 0.7) compared to moderate (λ_1 = 0.8) or high (λ_1 = 0.9) failure rates. This underscores the dual role of failure and repair processes: while higher repair efficiency mitigates the impact of failures, the inherent frequency of failures still constrains system performance. In other words, repair rate improvements cannot fully compensate for very high failure intensities, which highlights the importance of joint optimization of both reliability (failure prevention) and maintainability (repair efficiency) in engineering systems.

Figures 10 and 11 extend the analysis by examining the effect of repair rate on the busy periods of the repairman (Busy Period 1 and Busy Period 2). As expected, the results show that increasing repair rate decreases the length of repairman busy periods, as units are restored more quickly, reducing congestion in the repair facility. Interestingly, the busy periods tend to be higher when the failure rate is low ($\lambda1$=0.7) compared to higher failure rates. This somewhat counterintuitive result arises because, at lower failure rates, units fail less frequently but remain longer in repair when repair is slow, thereby stretching repair durations. Conversely, when failure rates are high, the system experiences more frequent failures, but with a higher repair rate, the turnaround is faster, balancing the repairman workload. These findings carry several important implications for system design and management. First, they confirm that enhancing repair rate is a highly effective strategy for improving availability, profitability, and lifetime, as well as reducing repair resource utilization. Second, they highlight that system designers should not rely solely on maintainability improvements; rather, a balanced approach that reduces component failure rates (e.g., through preventive maintenance, design improvement, or better operating conditions) while simultaneously improving repair efficiency will yield the greatest dependability and cost-effectiveness. Third, the observed relationship between failure intensity and repairman busy periods offers valuable insights into workforce and resource planning: even in systems with relatively low failure rates, inadequate repair speed can create bottlenecks in maintenance operations. From a broader reliability engineering perspective, these results reinforce the established understanding that system performance is strongly dependent on the interplay between failure mechanisms and repair strategies. They also demonstrate the necessity of integrated reliability and maintainability modeling, particularly in critical infrastructures such as power systems, telecommunication networks, and irrigation pumping systems, where both operational uptime and economic sustainability are key decision drivers.

5.0 CONCLUSION

This study evaluated the reliability performance of a standby three unit identical system in which two units operate actively while the third serves as a standby unit. By adopting a continuous time Markov chain framework, Explicit expressions were derived for key reliability indices, including mean time to failure (MTTF), and steady-state availablity, Busy priod and profit analysis. Numerical experiments and simulations were conducted to assess the system's behavior under varying failure and repair rates. The results consistently show that higher failure rates reduce availability, profit, and MTTF, while simultaneously increasing the busy periods of the repairman. Conversely, higher repair rates significantly enhance availability, profit, and MTTF, while reducing busy periods, thus demonstrating the critical role of maintainability in mitigating the adverse effects of frequent failures. Furthermore, the findings confirm that repair efficiency cannot entirely offset very high failure intensities, underscoring the need for a balanced strategy that improves both component reliability and repair mechanisms. Overall, the study highlights the dependability and economic benefits of incorporating standby redundancy in multi-unit systems. The insights provide a useful basis for system designers and decision makers in mission-critical applications – such as power, telecommunication, and irrigation infrastructures – where minimizing downtime and maximizing cost-effectiveness are essential.

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